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Computation of Proper and Improper Modes in Multilayered Bianisotropic Waveguides

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Abstract—**An efficient numerical method is presented to determine the loci of both the proper and complex improper modes of a multilayered bianisotropic planar waveguide. The propagation constants of the waveguide modes are expressed in terms of the zeros of a specific analytic function. The use of appropriate integration zero-searching methods is proposed since information about the possible number of complex improper modes cannot be previously extracted. The general formulation presented here has been applied to the study of the complex improper modes of single and two-layer structures containing magnetized ferrites. It has been found that the transition from physical proper to complex improper modes is made throughout a nonphysical real improper mode.**

I. INTRODUCTION

The grounded multilayered planar waveguide is the basic background of microstrip antennas, microstrip patch resonators and open dielectric waveguides for integrated optics and millimeter-wave integrated circuits [1], [2]. A topic which demands recent and increasing interest is the effect of an increasing number of layers [3] and substrate complexity [4] on the radiation pattern in antennas, the resonant frequency of patch resonators and the propagation characteristics in open dielectric waveguides. Computation and further analysis of the Green's function of the involved configuration can become essential. This analysis is usually carried out by studying the singularities of the Green's function: the branch-point singularities account for the free dipole radiation and the pole singularities for the background radiation and guided modes [1]. Thus, finding the pole singularities, which are located on a two-sheeted Riemann surface, is a preliminary step in obtaining closed-form representations of the Asymptotic Green's Function (AGF) [3], [5]. Assuming that the upper sheet of the Riemman surface is defined as fulfilling the radiation condition, the poles located on this sheet (*proper* sheet) form a finite and real subset which corresponds to the *bounded* modes guided by the layered slab. On the other hand, the complex and infinite subset of poles located on the bottom (*improper*) sheet, correspond to unbounded modes which are usually called *leaky* modes [1].

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There are some works in the literature devoted to the computation (and further application) of the proper and real improper modes [6]. Complex improper modes are treated in [5], where the possible significance of these complex modes is also discussed. Nevertheless, to our knowledge the substrate of the considered structures was assumed to be isotropic. Thus, the purpose of the present paper is to provide an efficient numerical method to determine the location of proper and improper waveguide modes in a planar waveguide with layered bianisotropic substrate. The method is based on the computation of the zeros of a specific *analytic* complex function (with no poles or branch-cut singularities). The search for the zeros of this function is carried out using an integral scheme which enables analysis of the complex plane (determining the number of zeros included within the closed integration contour) and accurate computation of the zeros.

II. ANALYSIS

In this section, the dispersion relation of a bianisotropic layered waveguide will be obtained. Note that this waveguide ranges from a simple grounded/covered/slab dielectric waveguide to waveguides with gyrotropic (semiconductor and/or ferrites biased by an arbitrarily oriented external d.c. magnetic field) and/or chiral layered substrate. The theory presented here is also applicable to those multilayered planar waveguides whose upper and bottom boundary conditions can be expressed as impedance or admittance dyads.

This work pointedly formulates the dispersion relation of the generic waveguide under consideration in terms of the zeros of an *analytic* function. This fact will be relevant in connection with the zero-searching procedure since the usual methods work efficiently when applied to analytic functions in the search region. The following dependence of the electromagnetic field at the plane of the interfaces (that is, the x, z components) $\mathbf{X} = (E_x, E_z, H_x, H_z)$, is assumed: $\mathbf{X}(x, y, z, t) = \exp(-j\omega t)\exp(-j\mathbf{k}_t \cdot \boldsymbol{\rho})\mathbf{X}(y)$, where ω is the angular frequency, $\boldsymbol{\rho} = x\mathbf{a}_x + z\mathbf{a}_z$ and $\mathbf{k}_t = k_x\mathbf{a}_x + k_z\mathbf{a}_z$ is the wavevector. As is shown in [7], the $\mathbf{X}$ vector inside each layer (denoted by the subscript i) is given in terms of a certain exponential matrix and a certain reference value, that is: $\mathbf{X}_i(y) = \exp(j\omega[\mathbf{Q}]_i y) \cdot \mathbf{X}_i(0)$. The explicit form of each element of the (4×4) $j\omega[\mathbf{Q}]_i$ matrix as a function of the layer characteristics is shown in [8].

Once fields at the upper interface of each layer are expressed in terms of fields at the bottom interface of each layer, we can express the field at the upper interface of the whole waveguide, $\mathbf{X}^u$, in terms of the field at the bottom interface of the waveguide, $\mathbf{X}^b$, by applying the continuity condition of the $\mathbf{X}$ vector at each intermediate interface. Thus, assuming that N_l is the total number of layers, the following matrix relation is obtained: $\mathbf{X}^u = [\mathbf{A}] \cdot \mathbf{X}^b$, where the $[\mathbf{A}]$ matrix is given by $[\mathbf{A}] = \prod_{i=1}^{N_l} \exp(j\omega[\mathbf{Q}]_i h_i)$ — h_i is the height of the i-th layer —.

The above matrix relation, together with the matrix impedance relations of $\mathbf{E}_t = (E_x, E_z)$ and $\mathbf{H}_t = (H_x, H_z)$ at the upper and bottom interfaces of the waveguide, enables writing the following matrix equations in terms of the $(2 \times 2)[\mathbf{A}_{ij}]$ submatrices of the $[\mathbf{A}]$ matrix and the impedance matrices, $[\mathbf{Z}_u]$ and $[\mathbf{Z}_b]$:

$$\mathbf{E}_t^u = [\mathbf{A}_{11}] \cdot \mathbf{E}_t^b + [\mathbf{A}_{12}] \cdot \mathbf{H}_t^b \tag{1}$$
$$\mathbf{H}_t^u = [\mathbf{A}_{21}] \cdot \mathbf{E}_t^b + [\mathbf{A}_{22}] \cdot \mathbf{H}_t^b \tag{2}$$
$$\mathbf{E}_t^u = [\mathbf{Z}_u] \cdot \mathbf{H}_t^u \tag{3}$$
$$\mathbf{E}_t^b = [\mathbf{Z}_b] \cdot \mathbf{H}_t^b. \tag{4}$$

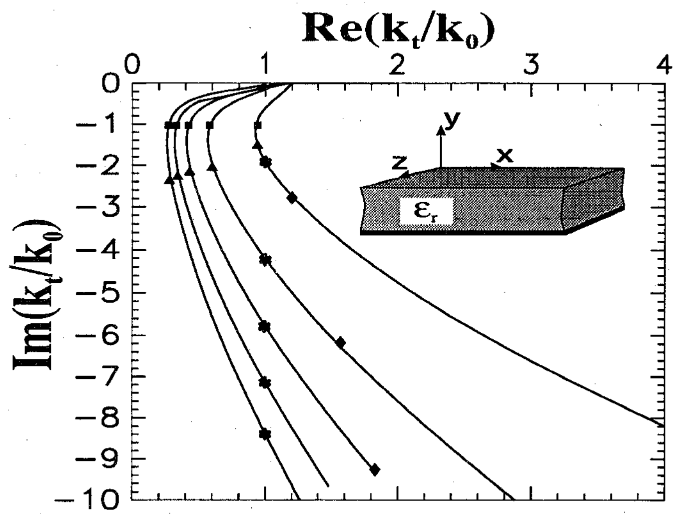


Fig. 1 Loci of the TE normalized propagation constant of the improper modes (TE leaky wave poles) in the grounded dielectric waveguide analyzed in Fig. 6 of [5], $\epsilon_r = 9$. (———): This work; (□, ∗, △, ◇):[5].

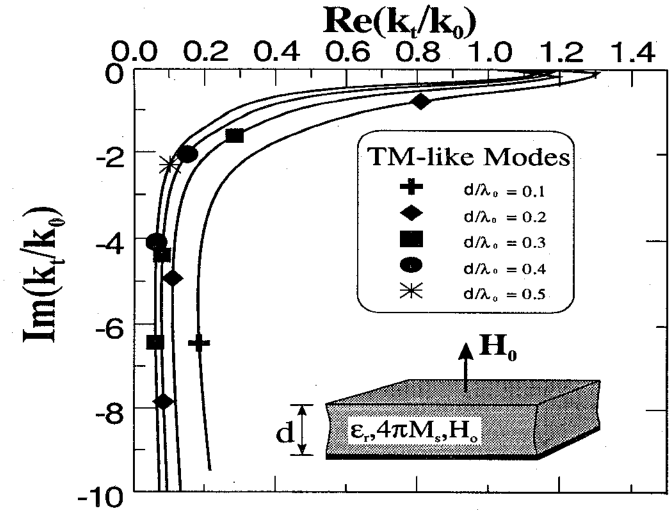


Fig. 2 Loci of the normalized propagation constant of the improper TM-like modes for a grounded waveguide with a ferrite substrate biased by a normally applied d.c. magnetic field. $\epsilon_r = 15$, $4\pi M_s = 1200$ G., $H_0 = 500$ Oe.

This (4 × 4) matrix equation may be solved providing that the determinant of the above matrix is null, and then the dispersion relation of the waveguide is given by:

$$F(k_x, k_z, \omega) = \det\{([\mathbf{A}_{11}] \cdot [\mathbf{Z}_b] + [\mathbf{A}_{12}]) - ([\mathbf{Z}_u] \cdot [\mathbf{A}_{21}] \cdot [\mathbf{Z}_b] + [\mathbf{Z}_u] \cdot [\mathbf{A}_{22}])\} = 0. \quad (5)$$

It should be emphasized that the present formulation of the waveguide dispersion relation leads to function $F(k_x, k_z, \omega)$ containing no poles. This statement can be justified on the basis of physical arguments although this can be also understood from the mathematical nature of the impedance matrices and construction of the $[\mathbf{A}]$ matrices. Nevertheless, function F may have branch points at $k_t = \pm k_0$ which would stem from the vacuum impedance matrices.

For a given frequency and fixed values of, for example, k_x, the propagation constants of the waveguide under study are given by the different values of k_z which make the function F vanish. As previously mentioned, the efficiency of root searching methods is closely related to the use of an analytic function. In the present case, the function $F(k_z; k_x, \omega)$ (k_x and ω are now considered as parameters) is not analytic in the complex k_z plane. Nevertheless, upon introducing a new complex variable z via the transformation

$$k_z = \sqrt{\omega^2 \epsilon_0 \mu_0 - k_x^2}\ \sin z, \quad (6)$$

the branch point disappears in the new complex z-plane (although poles appear in the multiples of $\pm\pi/2$). The transformation (6) maps the two-sheeted Riemann surface defined in the k_z-plane into regions of width 2π in the z-plane (more details on the above mapping can be found in [1]). Therefore, the dispersion relation of the waveguide can be expressed in analytic form in the z-plane as: $\left(z \pm \frac{\pi}{2}\right) F(z; k_x, \omega) = 0$.

The search for the zeros should be then carried out in the complex z-plane, using the integral method presented in some detail in [7]. Once all the z_m zeros inside a given region have been computed, the propagation constants of the waveguide are given by $k_{z,m} = \sqrt{\omega^2 \epsilon_0 \mu_0 - k_x^2}\ \sin z_m$.

III. Numerical Results

A computer code has been implemented to obtain the propagation characteristics and field patterns of the proper and improper modes of a grounded layered waveguide. First for comparison, the grounded dielectric slab previously studied in Fig. 6 of [5] is analyzed. Fig. 1 shows the loci of the normalized complex propagation constants corresponding to five TE improper modes of that structure. A good agreement is found between our results and the points marked in this figure, which reproduce the numerical values reported in the caption of Fig. 6 in [5].

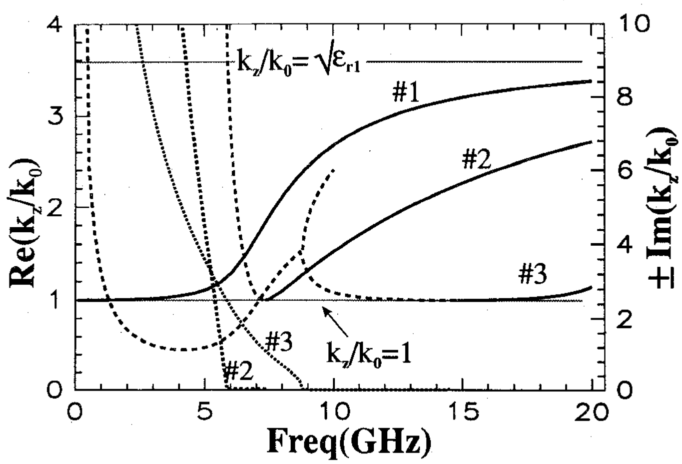
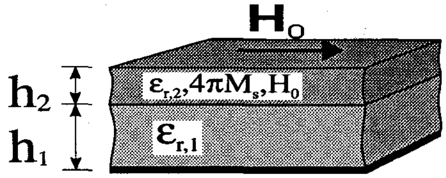


Fig. 3 Dispersion relation of the three slab-guided modes appearing in Table I. (————): $\mathrm{Re}(k_z/k_0)$ corresponds to proper modes. (– – – – – – –): $\mathrm{Re}(k_z/k_0)$ corresponds to improper modes. (············): $\pm\mathrm{Im}(k_z/k_0)$corresponding to improper modes.

After this comparison, novel results are now presented for open waveguides containing ferrite layers. Thus, Fig. 2 shows the loci of several TM-like modes in the complex (k_t/k_0)-plane for a grounded waveguide with a ferrite substrate biased by a normally applied d.c. magnetic field $\mathbf{H}_0$ (all the biased ferrite substrates are assumed fully saturated and H_0 represents the intensity of the internal $\mathbf{H}_0$ field). This figure shows that the effect of the leaky waves becomes more significant as d/λ_0 increases. Table I shows the propagation constants of the real proper modes, and several real and complex improper modes at a specific operation frequency, for the background waveguide proposed in [4]. This waveguide is composed of a biased ferrite (parallel to the interface) on a grounded isotropic substrate. The values presented in Table I refer to the propagation constant along the $+z$ direction and were obtained assuming $k_x = 0$. It can be seen in Table I that three slab-guided modes (proper modes) exist at 20

TABLE I
NORMALIZED PROPAGATION CONSTANTS OF THE PROPER AND VARIOUS IMPROPER MODES ($0 < \mathrm{Im}(k_z/k_0) < 11$, $0 < \mathrm{Re}(k_z/k_0) < 5$) FOR A GROUNDED WAVEGUIDE WITH A DIELECTRIC-FERRITE COMPOSITE SUBSTRATE. LOWER DIELECTRIC LAYER WITH h_1=2mm, h_2=1mm, AND BIASED FERRITE LAYER ($\mathbf{H}_0 = H_0\mathbf{a}_x$) WITH $\epsilon_{r,1} = 12.9$, $\epsilon_{r,2} = 12.6$, $4\pi M_s = 2750$ G, $H_0 = 8$ 25 Oe, Freq = 20 GHz.



Proper Modes

	$\mathrm{Re}(k_z/k_0)$
# 1	3.3705
# 2	2.7107
# 3	1.1443

Improper Modes

	$\mathrm{Re}(k_z/k_0)$	$\mathrm{Im}(k_z/k_0)$
# 1	3.3527	0
# 2	0.0711	-10.6579
# 3	0.0748	-7.9868
# 4	0.0923	-5.1117
# 5	0.4336	-1.2679
# 6	0.8296	-9.2161
# 7	1.1063	-4.4223
# 8	1.1500	-1.0485
# 9	1.1794	-10.7319
# 10	1.4345	-6.8996

GHz. The dispersion curves of these modes for frequencies below 20 GHz are shown in Fig. 3. The solid lines represent its frequency behavior when the corresponding root lies on the proper sheet; the dashed and dotted lines represent its mathematical prolongation on the improper sheet. If the *cutoff frequency* is defined as that frequency where the root of one mode passes through the branch cut, Fig. 3 shows that the fundamental TM slab-guided wave (marked by #1) has no cutoff frequency, but the other two slab-guided modes show cutoff frequencies at 7.35 and 14.55 GHz, respectively. Note that in open structures, the cutoff frequency separates the nature of the mode into proper and improper, rather than into propagating and evanescent. If Fig. 3 is read from the right- to the left-hand side, we can observe how the proper real mode #3 becomes an improper real mode at its cutoff frequency. This improper real mode encounters another improper real mode coming from high values of $\mathrm{Re}(k_z/k_0)$ at 8.7 GHz, and these two modes come together to form a complex improper mode below this frequency. The transition between the physical bound and unbound modes (that is, the proper real and the leaky modes) is made throughout the nonphysical real improper mode. This type of conduct has been previously reported in the literature and is usually known as *spectral gap* [9]. The above behavior is also found for mode #2, but it appears beyond the limits of Fig. 3. Moreover, the scheme described for mode #3 is always found for all slab-guided modes of grounded layered waveguides.

IV. SUMMARY

This work presented an efficient numerical procedure to compute the propagation constants of both the proper and improper modes of a planar bianisotropic layered waveguide bounded by upper and bottom interfaces which can be simulated by impedance/admittance dyads. The dispersion relation of the waveguide has been posed, in a compact way, as the roots of a certain analytic (no poles or branch cuts) function, and integral techniques are suggested to search efficiently for these roots. The transition from proper to improper modes in a grounded waveguide containing a biased ferrite layer has been studied, and a spectral gap has been found in the prolongation from the bound mode to the leaky mode.

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A Note on the Mode Characteristics of a Ferrite Slab

Hung-Yu Yang

***Abstract*—Properties of guided-wave modes of a ferrite slab propagating in the direction transverse to the bias field are reexamined. Analytic results for the frequencies where magnetostatic and dynamic modes exist simultaneously are found. The method of eliminating the dynamic modes in the magnetostatic-wave operation is described. The formulas for the distinction of oscillatory and surface-wave modes are also derived.**

I. INTRODUCTION

Guided-wave properties of a ferrite slab have been studied extensively in the past, for example with a magnetostatic analysis [1]–[5] and with a full-wave analysis [6]–[10]. It has been found that microwave devices with ferrite slabs are capable of space-frequency selection of signals [11]. It has been well recognized that a ferrite

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